Q1. Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed v_s relative to the air, while the observer is receding from the source, moving in the opposite direction with speed v_o relative to the air. Calculate the Doppler shift z.

Q2. There exists a redshift when the contribution of matter and radiation to energy density is equal. It is denoted by z_{eq}. Find an expression for z_{eq} in terms of energy densities, ρ_m and ρ_r. Then calculate the numerical value for z_{eq} given

- Matter: $\Omega_{m,0}h^2 = 0.143$
- Radiation: $\Omega_{r,0}h^2 = 4.15 \times 10^{-5}$

Q3. Consider solutions to Friedmann Equation for the case $\Omega_k \neq 0$ and $\Omega_\Lambda=0$. Show that for:
i) $\Omega_k > 0$, at late times the scale factor a(t) increases linearly with time.
ii) $\Omega_k < 0$, the scale factor first increases and then decreases with time. Find an expression for a_{max} in terms of the density parameters.

Q4. Consider a flat (i.e., a k = 0, or a Euclidean) universe with scale factor given by a(t) = $a_0=e^{\chi t}$.
where a_0 and χ are constants.
(a) Consider two galaxies which at some time t_1 are separated by a physical distance l_p. At this time one galaxy emits a pulse of light in the direction of the other. If the light pulse is received at the second galaxy at time t_2, what is the redshift z? (Recall that 1 + z is the factor by which the wavelength or the period of the light wave is increased.)
(b) At what time t_2 does the second galaxy receive the light pulse? Your answer should depend only on one or more of the quantities c, a_0, χ,
l_p, and t_1, but not on z.

Q5. A distant galaxy emits a spectral line at a rest-frame wavelength of **500 nm**. An astronomer on Earth observes this line at **2500 nm** using an infrared telescope.

(a) What is the **cosmological redshift** z of the galaxy?

(b) Assuming a **flat universe** with Hubble constant H_0=70 km/s/Mpc and matter density $\Omega_{m,0}$ = 0.3, estimate the **comoving distance** to the galaxy.

(c) What is the **lookback time** to the emission of the light (i.e., how long ago was the light emitted)?

Q6. Interestingly, in some cosmologies angular distance, $d_A(z)$ is not a monotonically increasing function of z, but reaches a maximum value at some redshift z_{max} and then decreases with increasing redshift. What this means is that in some cosmologies objects of a given proper size D will subtend a minimum angle $\delta\theta$ on the sky at $z = z_{max}$. Find an expression for this minimum angle for an Einstein-de Sitter cosmology, z_{max} = 1.25. Find the value for this minimum angle subtended by a galaxy cluster of typical diameter D = 1 Mpc.

Q7. Estimate the present age of the universe in Gyrs for an Einstein-de Sitter Cosmology.

Q8. Continuing with what was discussed during the sessions about evolution of the scale factor a(t). We discussed dust and radiation dominated cosmologies. Show that the scale factor grows exponentially in case of constant Dark Energy (Cosmological Constant) and linearly for curvature dominated Cosmologies. Find the expressions.

Q9. Find relation between d_L and d_A . Discuss the limit $z<<1$.